

Atom-field entanglement in a bimodal cavity

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Abstract

We investigate some aspects of the dynamics and entanglement of bipartite quantum system (atom-quantized field), coupled to a third “external” subsystem (quantized field). We make use of the Raman coupled model; a three-level atom in a lambda configuration interacting with two modes of the quantized cavity field. We consider the far off resonance limit, which allows the derivation of an effective Hamiltonian of a two-level atom coupled to the fields. We also make a comparison with the situation in which one of the modes is treated classically rather than prepared in a quantum field (coherent state).

1 Introduction

Three interacting quantum systems may also be viewed as a bipartite system coupled to a third, “external” system. The Raman coupled model, introduced some years ago [1] constitutes an example of a simple, analytically solvable model involving three quantum subsystems; a three level atom coupled to two modes of the cavity quantized field. In the limit in which the excited atomic state is well far-off resonance, a simpler, effective two-level Hamiltonian may be derived, either by an adiabatic elimination of the upper atomic level [1] or via a unitary transformation [2]. In such a procedure, energy (Stark) shifts arise and care must be taken, given that they may not always be neglected [3]. In fact, the presence of the shifts normally leads to a very different dynamics, e.g., from a non-periodic to a periodic atomic inversion. Several features of the dynamics of such model have already been investigated [1]; in particular, quantum entanglement and possible applications to quantum computation [4, 5]. Here we are going to discuss a different aspect of that system; the influence of one of the fields (a cavity mode) on the dynamics of the atom as well as on the bipartite entanglement between the atom and the other cavity mode. We are going to consider different field preparations, such as coherent and thermal states, and we will also compare our results to the case in which one of the modes is not quantized, but treated as a classical field, instead. For simplicity, we do not take into account cavity losses or atomic spontaneous decay. Our paper is organized as follows: in Sec. II we present the model and solution. In Sec. III we discuss dynamical features with different preparations. In Sec. IV we present the evolution of bipartite entanglement, and in Sec. V we summarize our conclusions.

2 The model and solution

2.1 Two quantized modes

We consider a three level atom (levels 1,2,3) in interaction with two modes (mode 1, of frequency ω_1 and mode 2, of frequency ω_2) of the quantized field in a lambda configuration. Direct transitions between the lower levels 1 and 2 are forbidden. If the upper level (level 3) is highly detuned from the fields (detuning Δ), the effective Hamiltonian may be written as [2]

$$H_{eff} = \sigma_{11}E_1 + \sigma_{22}E_2 + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 - \hbar \frac{g_1^2}{\Delta} \sigma_{11} a_1^\dagger a_1 - \hbar \frac{g_2^2}{\Delta} \sigma_{22} a_2^\dagger a_2 - \hbar \frac{g_1 g_2}{\Delta} \left(a_1^\dagger a_2 \sigma_{12} + a_2^\dagger a_1 \sigma_{21} \right), \quad (1)$$

where σ_{12} and σ_{21} are the transition operators between levels 1 and 2, $a_1(a_1^\dagger)$ is the annihilation (creation) operator of mode 1, $a_2(a_2^\dagger)$ is the annihilation (creation) operator of mode 2, and $g_1(g_2)$ are the couplings of the transitions 1–3 and 2–3, respectively. The effective Hamiltonian is valid in the limit $g_1/\Delta(g_2/\Delta) \ll 1$, but under certain conditions, exact similar Hamiltonians may also be obtained [6]. The Stark shift terms $-\hbar \frac{g_1^2}{\Delta} \sigma_{11} a_1^\dagger a_1$ and $-\hbar \frac{g_2^2}{\Delta} \sigma_{22} a_2^\dagger a_2$ are usually neglected [1, 5], but one should be very careful, given that their inclusion results in a Rabi frequency depending linearly on the photon numbers n_1 and n_2 . As a

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consequence, because of the shifts, the dynamics of the Raman coupled model becomes basically periodic, with Rabi frequency

$$\Omega_{n_1, n_2} = \frac{[g_1^2 n_1 + g_2^2 (n_2 + 1)]}{2\Delta} \quad (2)$$

in contrast to the Rabi frequency, which is proportional to $\sqrt{n_1(n_2 + 1)}$, if the Stark shifts are neglected [1]. Assuming an initial density operator of the product form,

$$\rho(0) = \sum_{n_1 n_2 m_1 m_2}^{\infty} \rho_{n_1 n_2 m_1 m_2} |1; n_1, n_2\rangle \langle 1; m_1, m_2|, \quad (3)$$

i.e., with the atom initially prepared in level 1, and the fields in generic states characterized by the coefficients $\rho_{n_1 n_2 m_1 m_2}$. The full time-dependent density operator for the tripartite system may be written as

$$\begin{aligned} \rho(t) = & \sum_{n_1 n_2 m_1 m_2}^{\infty} \{A_{n_1 n_2 m_1 m_2} |1; n_1, n_2\rangle \langle 1; m_1, m_2| + B_{n_1 n_2 m_1 m_2} |2; n_1 - 1, n_2 + 1\rangle \langle 2; m_1 - 1, m_2 + 1| \\ & + C_{n_1 n_2 m_1 m_2} |1; n_1, n_2\rangle \langle 2; m_1 - 1, m_2 + 1| + h.c.\}, \end{aligned} \quad (4)$$

with coefficients

$$\begin{aligned} A_{n_1 n_2 m_1 m_2} &= \rho_{n_1 n_2 m_1 m_2} e^{i\nu_{n_1 n_2 m_1 m_2} t} k_{1, n_1, n_2} k_{1, m_1, m_2}^*, \\ B_{n_1 n_2 m_1 m_2} &= \rho_{n_1 n_2 m_1 m_2} e^{i\nu_{n_1 n_2 m_1 m_2} t} k_{2, n_1, n_2} k_{2, m_1, m_2}^*, \\ C_{n_1 n_2 m_1 m_2} &= \rho_{n_1 n_2 m_1 m_2} e^{i\nu_{n_1 n_2 m_1 m_2} t} k_{1, n_1, n_2} k_{2, m_1, m_2}^*, \end{aligned}$$

where

$$\nu_{n_1 n_2 m_1 m_2} = (m_1 - n_1) \left(\omega_1 - \frac{g_1^2}{2\Delta} \right) + (m_2 - n_2) \left(\omega_2 - \frac{g_2^2}{2\Delta} \right), \quad (5)$$

and

$$\begin{aligned} k_{1, n_1, n_2}(t) &= \cos(\Omega_{n_1, n_2} t) + i \left[\frac{n_1 - r^2 (n_2 + 1)}{n_1 + r^2 (n_2 + 1)} \right] \sin(\Omega_{n_1, n_2} t), \\ k_{2, n_1, n_2}(t) &= \frac{2ri\sqrt{n_1 (n_2 + 1)}}{[n_1 + r^2 (n_2 + 1)]} \sin(\Omega_{n_1, n_2} t), \end{aligned}$$

where the Rabi frequency Ω_{n_1, n_2} is given in Eq. (2) and having defined $r \equiv g_2/g_1$.

2.1.1 Atomic dynamics with different initial field preparations

The atomic response to the fields may be characterized by the atomic population inversion as a function of time, or

$$W(t) = 2 \text{Tr} [\rho(t) |2\rangle \langle 2|] - 1 = 8 \sum_{n_1, n_2}^{\infty} p_{n_1} p_{n_2} \frac{r^2 n_1 (n_2 + 1)}{[n_1 + r^2 (n_2 + 1)]^2} \sin^2(\Omega_{n_1, n_2} t) - 1, \quad (6)$$

where $p_{n_1} p_{n_2} = \rho_{n_1 n_2; n_1 n_2}$ is the product of the photon number distributions of the initial fields. The atomic inversion has peculiar features depending on the field statistics. For instance, in another well known model of optical resonance having a two level atom coupled to a single mode field, the Jaynes Cummings model (JCM), a field initially in a Fock state leads to pure oscillations of the atomic inversion, while an initial coherent state causes collapses and revivals [7] of the Rabi oscillations. On the other hand, in the JCM, for a field initially in a thermal state, the structure of collapses and revivals becomes highly desorganized [8] and look like random. In the Raman coupled model, in which an effective two level atom

is coupled to two fields, rather than one, we expect of course a different behaviour. Perhaps the most striking difference to the JCM is the periodicity of the atomic inversion; moreover, the “revival” times do not depend on the intensity of the fields, and the field statistics has only little effect on the pattern of collapses and revivals [3]. In order to illustrate this, in Fig. (1) we have plots of the atomic inversion as a function of the scaled time $\tau = g_1 t$ for modes 1 and 2 prepared in coherent states: $p_{n_i} = e^{-\bar{n}_i} \bar{n}_i^{n_i} / n_i!$ [Fig. 1(a)] and for mode 1 prepared in a coherent state and mode 2 prepared in a thermal state: $p_{n_1} = e^{-\bar{n}_1} \bar{n}_1^{n_1} / n_1!$ and $p_{n_2} = \bar{n}_2^{n_2} / (\bar{n}_2 + 1)^{n_2+1}$ [Fig. 1(b)]. We may note the periodical and well defined revivals occurring at the same times in both cases. Curiously, even in the case of a thermal (chaotic) initial preparation, revivals are well defined, although their amplitude is slightly suppressed; probably an effect due to the broader thermal distribution. We also note that if one of the fields is prepared in a Fock state, if the other mode is prepared in a coherent state, for instance, collapses and revivals will still occur, due to the presence of the “in phase” and “out of phase terms” in Eq. (6). In Fig. (2) we have a plot of the atomic inversion for mode 1 prepared in a N photons Fock state and mode 2 prepared in a coherent state, and we note again the pattern of periodic and regular collapses and revivals. In what follows, this particular preparation (Fock-coherent) is going to be compared with a situation termed “partially classical”, having one of the fields, mode 2, treated classically rather than prepared in a coherent state (fully quantized case).

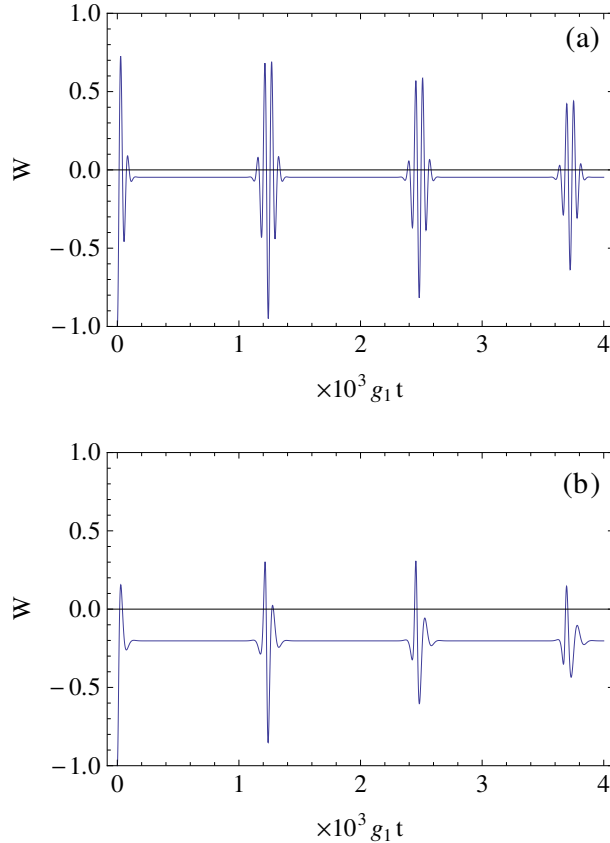


Figure 1: Atomic inversion as a function of the scaled time $\tau = g_1 t$ for a) modes 1 and 2 prepared in coherent states and b) mode 1 prepared in a coherent state and mode 2 prepared in a thermal state. In both cases $\bar{n}_1 = 10.5$ and $\bar{n}_2 = 10.1$. We have considered $g_1 = g_2$ and $r = 1.012$.

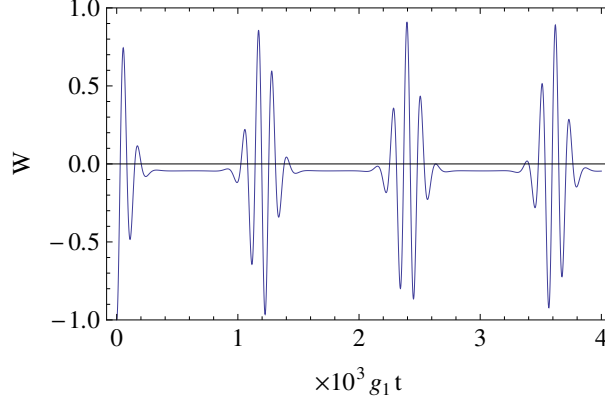


Figure 2: Atomic inversion for mode 1 prepared in Fock state ($N = 5$) and mode 2 prepared in a coherent state ($\bar{n} = 5$), with $r = 1.023$.

2.2 One quantized mode

Now we consider mode 2 as being a classical field of amplitude Ω_L ; the “partially classical” case, keeping mode 1 quantized. The effective Hamiltonian in the far-off resonance limit for the excited state is, in this case [9],

$$H'_{eff} = \frac{g^2 a^\dagger a}{\Delta} \sigma_{11} + \frac{g^2 r'^2}{\Delta} \sigma_{22} + \lambda (\sigma_{12} a^\dagger + \sigma_{21} a), \quad (7)$$

where g is the quantum field/atom coupling constant, $\lambda = g |\Omega_L^*| / \Delta$ is the effective coupling constant and $r' = |\Omega_L^*| / g$. Note that apart from some shifts, the effective Hamiltonian is very similar to a JCM Hamiltonian. This means that the atomic response to a field (mode 1) prepared in a Fock state is going to be in the form of pure Rabi oscillations, in contrast to the case where mode 2 is a quantum field prepared in the “quasi-classical” coherent state, which shows collapses and revivals. This is another example showing that regarding the atomic response to a field, an intense quantum coherent state is not equivalent to a classical field.

3 Entanglement

We would like now to discuss the entanglement between the atom and one mode of the field (mode 1, for instance) considering the other mode as an “external” coupled sub-system. We then trace over the variables of mode 2, initially prepared in a coherent state, and examine the degree of entanglement between the atom and the remaining field (mode 1) initially prepared, for the sake of simplicity, in a Fock state. In order to quantify entanglement, we use the negativity. As we have done in the case of the atomic inversion, we will compare the results with the case in which mode 2 is considered to be a classical field. For an initial state having the atom in level 1, mode 1 in a Fock state $|N\rangle$ and mode 2 in a coherent state $|\alpha\rangle$, i.e., a product state $|\psi(0)\rangle = |1\rangle \otimes |N\rangle \otimes |\alpha\rangle$, the joint atom-mode 1 density operator, obtained after tracing over mode 2 from Eq. (4), $\tilde{\rho}(t) = \text{Tr}_2[\rho(t)]$ is given by

$$\begin{aligned} \tilde{\rho}(t) = & \sum_{n=0}^{\infty} |c_n|^2 k_{1,N,n} k_{1,N,n}^* |1; N\rangle \langle 1; N| + \sum_{n=0}^{\infty} |c_n|^2 k_{2,N,n} k_{2,N,n}^* |2; N-1\rangle \langle 2; N-1| \\ & + F(t) \sum_{n=0}^{\infty} c_{n+1} c_n^* k_{1,N,n+1} k_{2,N,n}^* |1; N\rangle \langle 2; N-1| + h.c., \end{aligned}$$

with $F(t) = e^{-i\left(\omega_2 - \frac{g_2^2}{2\Delta}\right)t}$ and $c_n = e^{-\bar{n}/2} \alpha^n / \sqrt{n!}$.

We may then calculate the negativity as a function of time

$$\mathcal{N}(t) = 2r\sqrt{\bar{n}N} \left\{ \left[\sum_{n=0}^{\infty} e^{-\bar{n}} \frac{\bar{n}^n}{n!} \frac{[N - r^2(n+2)]}{[N + r^2(n+2)]} \frac{\sin(\Omega_{N,n}t) \sin(\Omega_{N,n+1}t)}{[N + r^2(n+1)]} \right]^2 + \left[\sum_{n=0}^{\infty} e^{-\bar{n}} \frac{\bar{n}^n}{n!} \frac{\sin(\Omega_{N,n}t) \cos(\Omega_{N,n+1}t)}{[N + r^2(n+1)]} \right]^2 \right\}^{1/2}, \quad (8)$$

where $\bar{n} = |\alpha|^2$ and $\Omega_{N,n} = [g_1^2 N + g_2^2(n+1)]/2\Delta$. It is also worth to compare the negativity to the linear entropy relative to the atomic state (obtained after tracing over mode 1, $\rho_A(t) = \text{Tr}_1[\tilde{\rho}(t)]$), which is defined as $\zeta_A(t) = 1 - \text{Tr}[\rho_A^2(t)]$ or

$$\zeta_A(t) = 2 \sum_{n=0}^{\infty} e^{-\bar{n}} \frac{\bar{n}^n}{n!} \frac{4r^2 N(n+1)}{[N + r^2(n+1)]^2} \sin^2(\Omega_{N,n}t) - 2 \left\{ \sum_{n=0}^{\infty} e^{-\bar{n}} \frac{\bar{n}^n}{n!} \frac{4r^2 N(n+1)}{[N + r^2(n+1)]^2} \sin^2(\Omega_{N,n}t) \right\}^2. \quad (9)$$

The linear entropy may be used as a measure of the quantum state purity, as it is zero for a pure state and > 0 for a mixed state. In Fig. (3) we have plotted the negativity (3a) and the linear entropy (3b) as a function of the scaled time $g_1 t$. We note that entanglement is maximum approximately in the middle of the collapse region, and the atom-mode 1 state tends to become separable at the revival times themselves (see Fig. 2). At the same time one may notice the similarities and differences between the negativity and the linear entropy; the maximum of entanglement coincides with the maximum of mixedness, and separability approximately occurs at times in which the atom is close to a pure state. However, we should point out that there are time intervals of maximum mixedness which do not correspond to maximum entanglement; this explicitly shows us the inadequacy of the linear entropy as a measure of entanglement (as expected), given that the state under consideration (atom-mode 1) is generally not a pure state. Again, the evolution of entanglement in the “partially classical” case is going to be very different. We would like to remark that in this case a bipartite quantum system is under the action of an external classical field, instead of a tripartite system from which we have obtained a bipartite system by tracing over one of the subsystems. Having mode 1 prepared in a N photon Fock state (mode 2 being a classical field), we have the following expressions for the atomic inversion (with the atom initially in state 1),

$$W'(t) = \frac{8r'^2 N}{(N + r'^2)} \sin^2 \left[\frac{(N + r'^2)}{2r'} \tau' \right] - 1, \quad (10)$$

and the negativity,

$$\mathcal{N}'(t) = \frac{2r'\sqrt{N}}{(N + r'^2)} \left\{ \sin^2 \left[\frac{(N + r'^2)}{2r'} \tau' \right] - \frac{4r'^2 N}{(N + r'^2)^2} \sin^4 \left[\frac{(N + r'^2)}{2r'} \tau' \right] \right\}^{1/2}, \quad (11)$$

where the scaled time is defined as $\tau' = g|\Omega_L^*|t/\Delta$ and $r' = |\Omega_L^*|/g$. Compared to the fully quantized situation; there are no collapses and revivals, as the atomic inversion and the negativity are now periodic functions of time. This is illustrated in Fig. (4), where we have plotted the atomic inversion in Eq. (10) and the negativity in Eq. (11) as a function of the scaled time τ' . For a convenient choice of the parameter r' , the atomic population completely inverts and returns to its initial state at times in which atom-mode 1 are in a separable state, a very different behaviour from what happens if mode 2 is a quantized field.

4 Conclusions

We have presented a study of the dynamics of three coupled quantum systems: one three level atom and two quantized cavity fields, focusing on some properties of the atomic system (population inversion) and the atom-mode 1 bipartite system (entanglement). The second field (mode 2) has been basically traced

out and treated as an “external” subsystem. Our study has been based on the Raman coupled model, involving an effective two level atom coupled to two electromagnetic cavity fields. We have considered different preparations for mode 2: coherent and thermal states of the quantized field, as well as a classical field. In order to keep the consistency of the effective Hamiltonian, as has been already pointed out [3, 4] we have retained the Stark shift terms in the Raman Hamiltonian, which have the remarkable effect of keeping the dynamics periodic. Moreover, the periodic revival times will not depend on the statistics of the fields. This means that having one mode (or even two) prepared in the (highly mixed) thermal state will not change the regular atomic response during the atom/fields interaction, as we have shown explicitly here. This is a nice example of dynamical features robust against substantial variations in the field statistics. We have addressed the issue of bipartite entanglement between the atom and mode 1, after tracing out mode 2: we have found that for mode 1 initially prepared in a Fock state and mode 2 in a coherent state, the atom/mode 1 reach maximum entanglement in the collapse region of the atomic inversion. We have also compared both the atomic response and entanglement in the case in which mode 2 is treated as a classical field (“partially classical” case), rather than a quantum coherent “quasi-classical” field, which result in very different evolutions.

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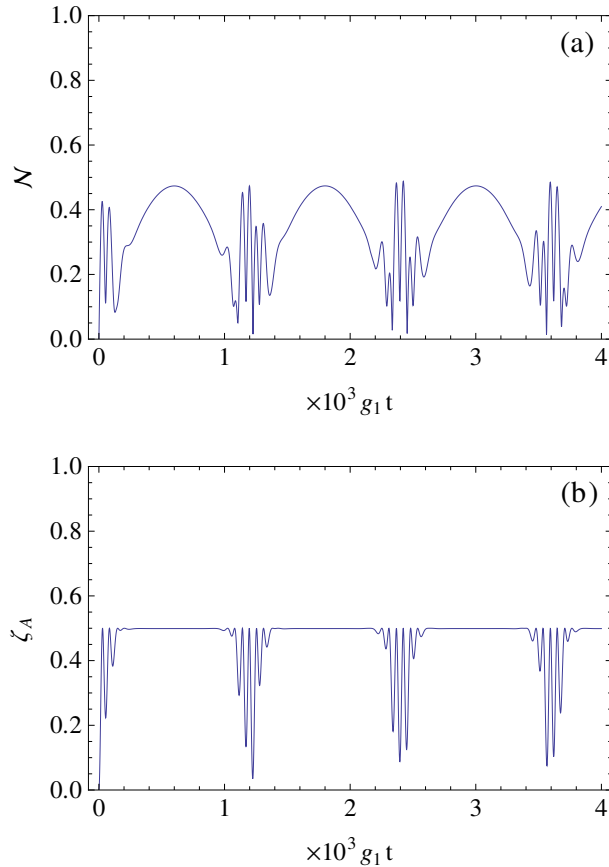


Figure 3: Negativity (a) and linear entropy (b) as a function of the scaled time $g_1 t$ for mode 1 prepared in Fock state ($N = 5$) and mode 2 prepared in a coherent state ($\bar{n} = 5$), with $r = 1.023$.

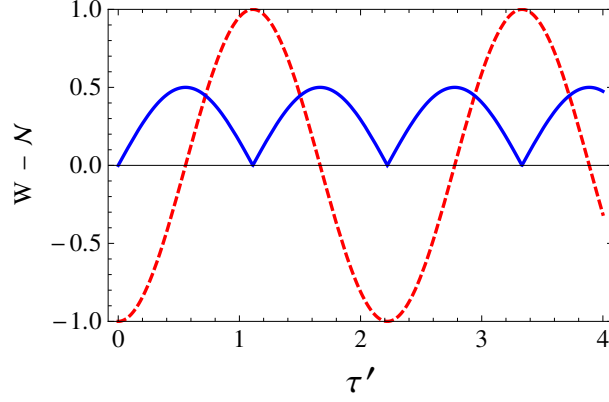


Figure 4: Atomic inversion (red, dashed curve) and the Negativity (blue, continuous curve) as a function of the scaled time τ' in the “partially classical” case, having mode 1 prepared in a Fock state ($N = 2$) and $r' = 1.41$.

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